First Steps in Updating Knowing How

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Overview of the talk

- Background
- The Knowing How logic
- Dynamic modalities: Ontic & epistemic updates
- Conclusions and future work

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- This work: a dynamic epistemic approach of knowing how.
  - Actions updating different kinds of information.

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Knowing How: Models

Definition (Uncertainty-based LTS)

An $\mathsf{LTS}^{\mathsf{U}}$ is a tuple $\mathcal{M} = \langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \{\mathbb{S}_i\}_{i \in \mathsf{Agt}}, \mathsf{V} \rangle$ where:

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$\mathbb{S}_i$ represents the sets of plans agent i cannot distinguish between each other.

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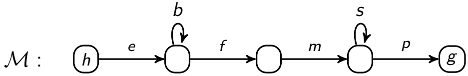

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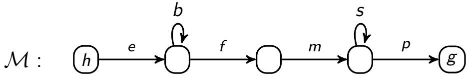

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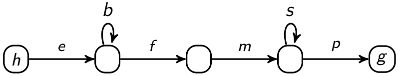

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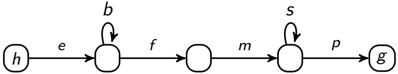

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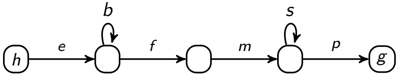

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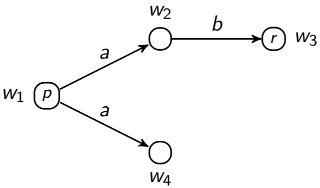

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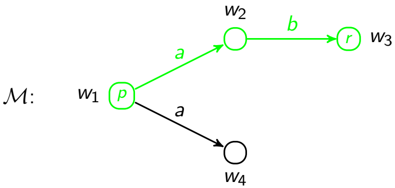

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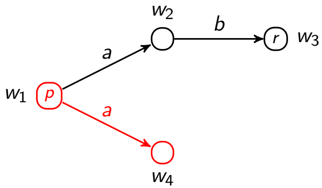

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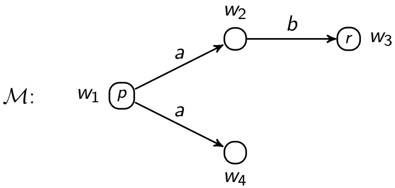

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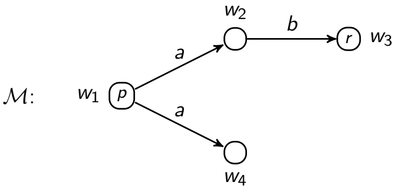

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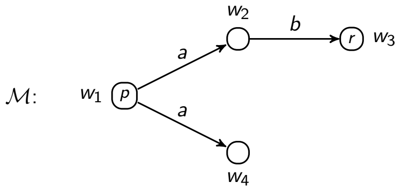

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- $\sigma \in \text{Act}^*$ is SE at a state u iff

  every partial execution of σ from u can be completed.
- $\pi \subseteq \text{Act}^*$ is SE at a state u iff for *all* $\sigma \in \pi$, σ is SE at u.

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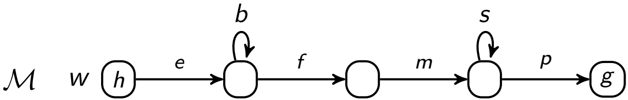

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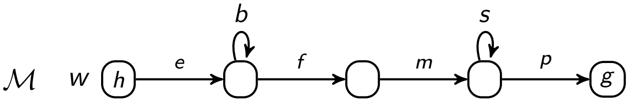

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This enables us to define ways of updating these two types of information.

Ontic updates: Announcement

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- $\mathsf{V}_{!\chi}(w) = \mathsf{V}(w)$.

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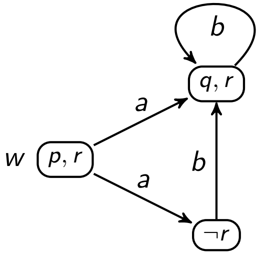

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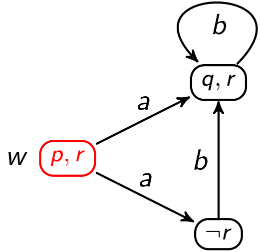

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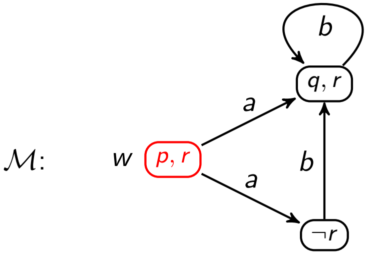

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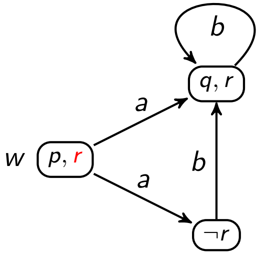

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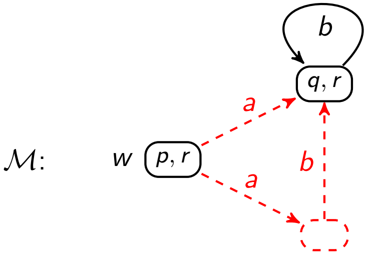

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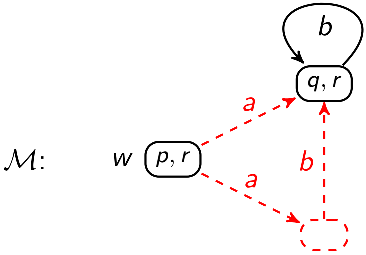

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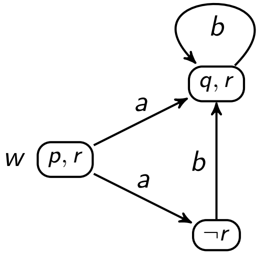

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Theorem

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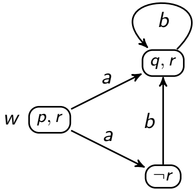

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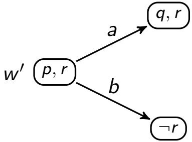

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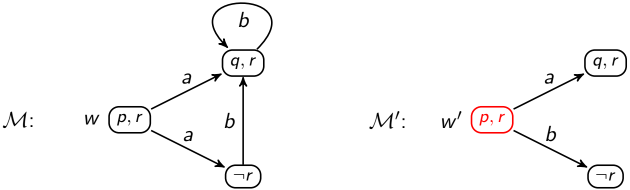

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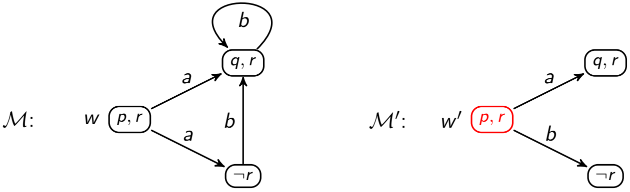

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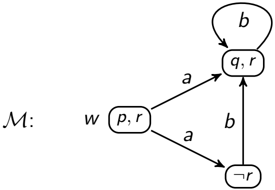

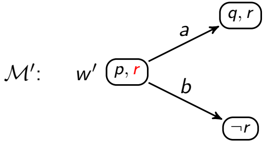

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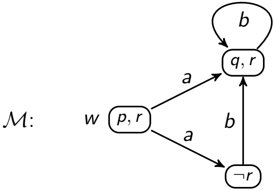

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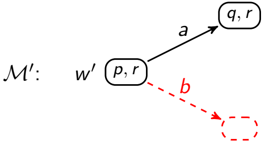

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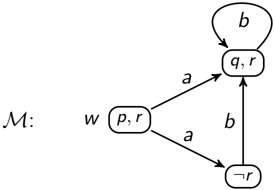

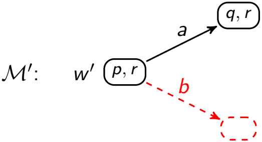

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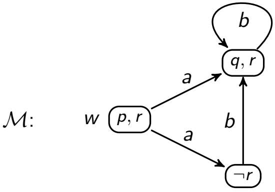

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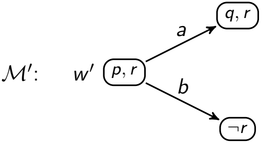

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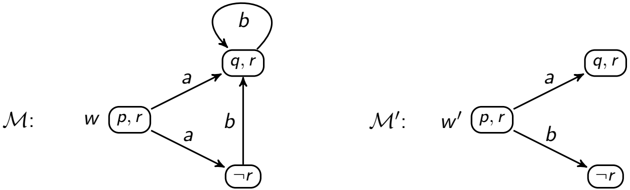

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Epistemic updates: Refinement (L_{Ref})

Definition (L_{Ref} formulas)

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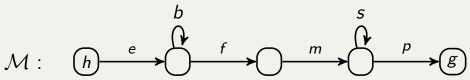

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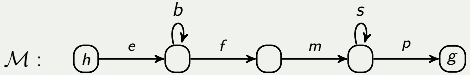

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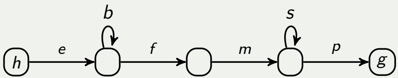

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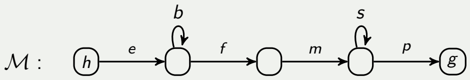

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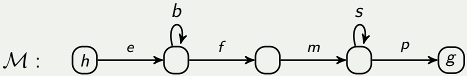

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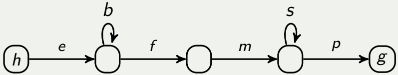

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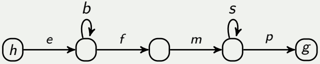

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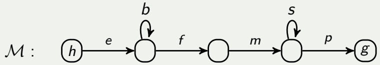

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L_{Lh} is more expressive than $\mathsf{L}_{\mathsf{Kh}_i}$.

Conclusions

Dynamic modalities in the context of *knowing how* logics.

- Ontic updates:
  - Announcement-like and arrow-update-like modalities
  - Axiomatizations over a particular class of models
- Epistemic updates:
  - Refining the perception of an agent regarding her own abilities.
  - Preliminary thoughts and some semantic properties.

Future work

- Study other dynamic operators in this context.
- Explore alternative techniques for obtaining proof systems without a general rule of substitution.
- Find fragments that are axiomatizable via reduction axioms by studying the operators' expressivity.